Risk-parity investing

Portfolio optimization in an uncertain world

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Outline

- 1. Challenging Markowitz
- 2. Risk parity investment strategy on equities
- 3. Risk parity investment strategy on corporate bonds NEW
- 4. Conclusion

Modern Portfolio Theory (MPT) revisited

Mean-variance efficient portfolios are risk optimal only if risk is foreseeable, i.e.

$$H_0: \quad R \sim N(\mu, \Sigma)$$ where Σ known with certainty.

Admitting uncertainty changes the perception.

Under uncertainty risk optimality has two determinants:

price variance ↓ diversification ↓

Generalised MPT: min. $x^T \Sigma\, x \; - \; \lambda \cdot x^T \, ln(x)$

where λ indicates degree of foreseeability / entropy.

Foreseeability / entropy

Generalised MPT: min. $x^T \Sigma\, x \;-\; \lambda \cdot x^T\, ln(x)$

1. No foreseeability – high entropy $(\lambda = \infty)$ – returns uniformly distributed

   optimum: equal weighting (1/*N*)

2. Intermediate level

   optimum: risk parity (1/risk contribution)

3. High foreseeability – low entropy $(\lambda = 0)$ – returns multivariate normal

   optimum: minimum variance

Generalised Modern Portfolio Theory

Risk optimality is defined by

price variance – efficient positioning with respect to foreseeable risk

diversification – safety net, or protection, against unforeseeable risk

Relative importance determined by degree of foreseeability, which is partly

objective – predictability of covariance varies between/within asset classes

subjective – how much diversification (safety) to forego for risk efficiency ?

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Coherence with generalised MPT

Coherent investment strategies

1. Top-down approach*

Since covariance levels are more stable/predictable within asset classes than between, Markowitz optimization tends to be deployed within- and risk budgeting between asset classes.

2. Bridgewater® All Weather Funds

Four equally-probable economic scenarios are weighted by their respective volatility levels.

3. Global Macro

Asset class weights set inversely proportional to volatility levels

* Discussed by Scherer (2007)
Portfolio Construction & Risk Budgeting

Coherence with generalised MPT

Incoherent investment strategies

1. Equal Risk Contribution* (ERC)

If one privileges risk parity, deliberately foregoing risk efficiency, the intention is to seek protection against shocks he cannot foresee. It is incoherent to admit uncertainty and use Σ unthoughtfully in the ERC calculations.

2. Maximum diversification*

Note the reliance on Σ in the definition of diversification: $\dfrac{\sum_i x_i \sigma_i}{\sqrt{x^T \Sigma\, x}}$

Defining diversification on (foreseeable) risk is incoherent.

* Qian (2006) Risk Parity and Diversification

* Choufaeity and Coignard (2008) Toward Maximum Diversification

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Toward coherent risk parity

Risk parity is the risk-optimal investment approach in an uncertain world.

Before optimising a portfolio one should

1. define the degree of foreseeability / uncertainty / level of entropy
2. specify a risk model you consider <u>certain</u>

We examine two postulates:

1. one notch down from darkness

   only volatility levels are predictable

2. two notches down from darkness:

   Capital Asset Pricing Models holds

   price behaviour effectively captured by market factor plus specific risk

Toward coherent risk parity

Postulate 1. Volatility levels (σ) predictable

Optimum at $x_i = {}^{1}/_{\sigma_i}$

minimum variance $\sigma_p{}^2 = \sum_i (x_i \sigma_i)^2 = 1$

maximum diversification - low portfolio concentration

Postulate 2. CAPM – market betas (β) and specific volatilities (σ) predictable

Optimum* when $x_i \propto {}^{1}/_{\sigma_i}$ & $x_i \propto {}^{1}/_{\beta_i}$

* Clarke et al. (2013) derive the near closed-form solution

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Coherent risk parity investing*

Postulate 2[bis]. constrained version of CAPM holds

$$\forall_{i \in industry\ j}: \quad R_{it} = \beta_j \cdot {R_t}^M + \varepsilon_{it}$$

Optimum when $x_i = {}^{1}/_{\sigma_i}$ between individual assets

$x_j = {}^{1}/_{\beta_j}$ between industry groups

Two-step portfolio construction:

1. Build the investment target: reweigh market index weights by above rule
2. Build an investible portfolio that tracks the target

* Russo (2014)

Back-tests

equities & corporate bonds

Testbed

- Region: Europe
- Period: 05/2007 to 06/2016
- Universe: MSCI Europe & Barclays Euro Corporate
- Industries: MSCI GICS and Barclays Level3 aggregated to ten groups
- Monthly data frequency

- 90% overlap between equity and bond universe
- Similar number of issuing firms: 447 for equities and 472 for bonds in 08/’16
- Similar sector breakdown

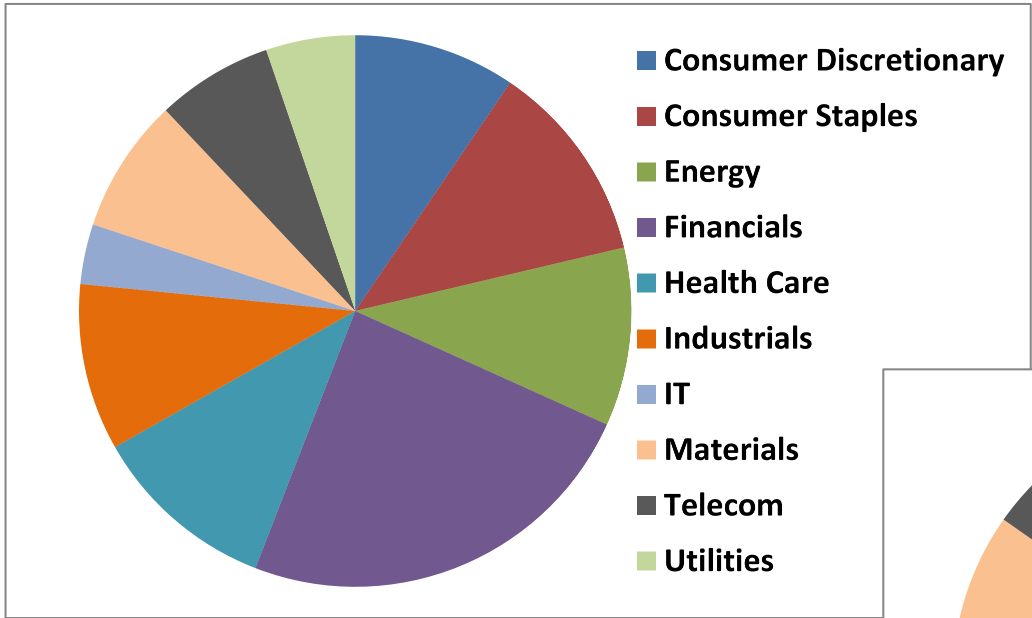
Test setup

- Portfolio rebalanced at regular intervals (no foresight)
- *Ex post* performance over test period compared with standard benchmark

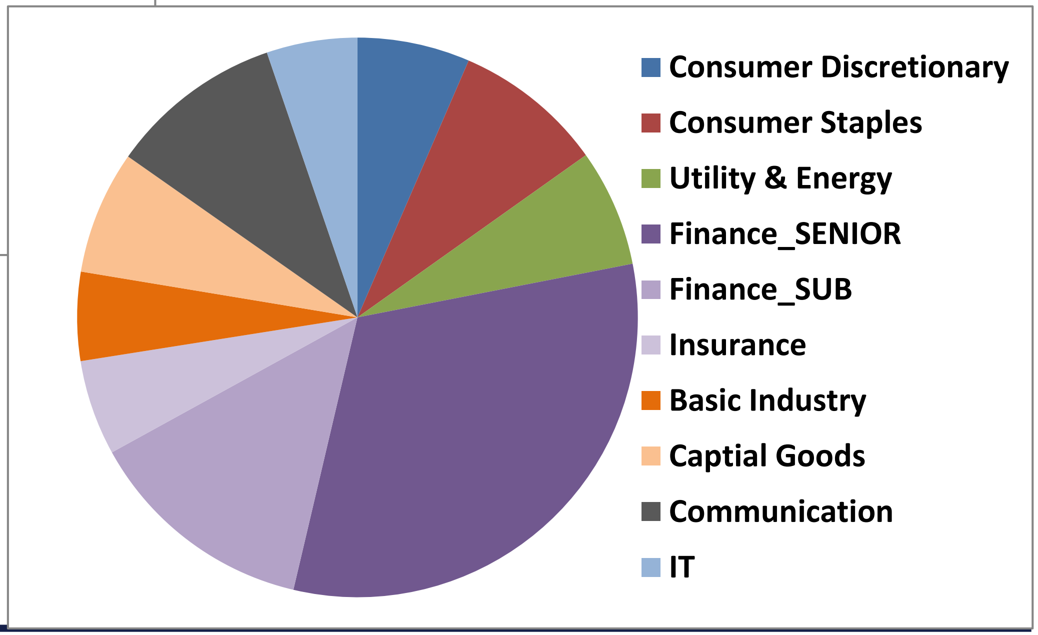
Testbed

Sector breakdown of European equity & bond market

Equity

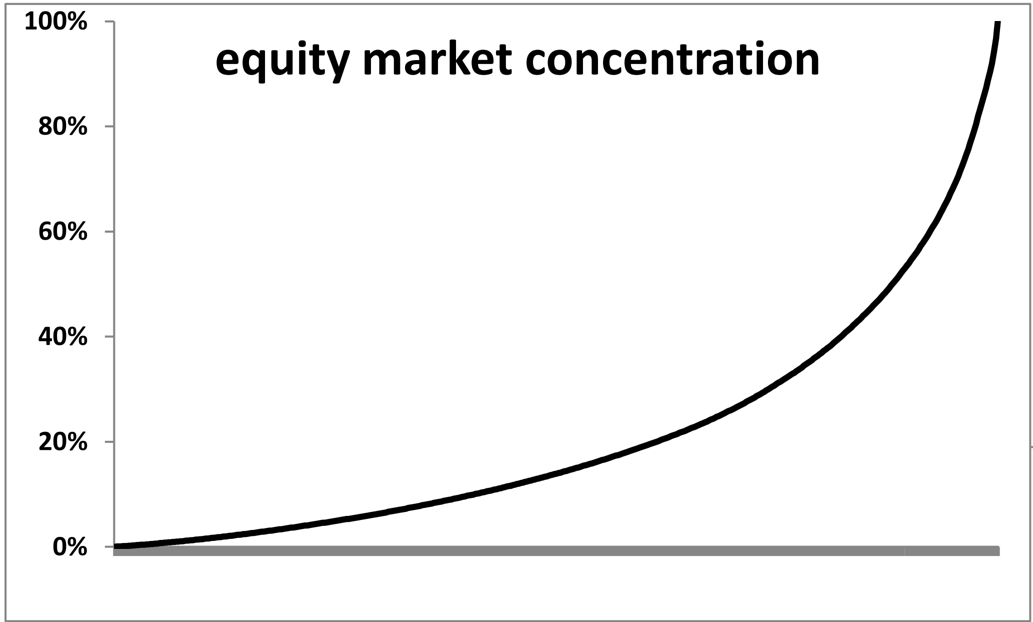


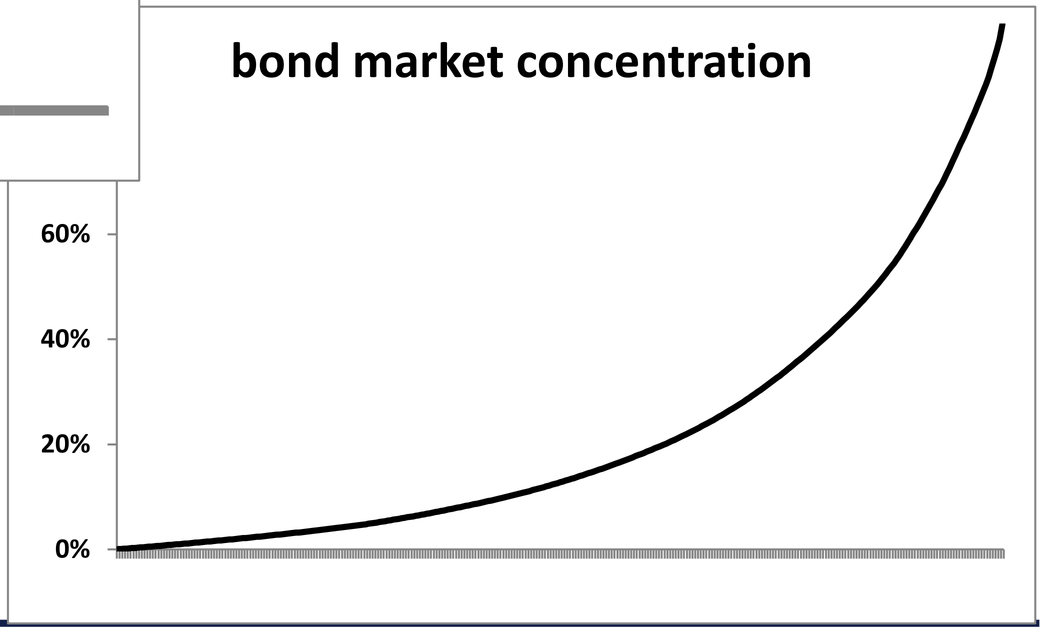
Bonds



Concentration of the equity & bond market in Europe

Lorenz curves





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Key test results

equity & corporate bonds

- Feasibility
  - Portfolios truly investible
  - Portfolio construction doable using easy-accessible data
- Serves the purpose: offers safe beta exposure
  - Lower volatility than benchmark for reaping essentially same premium
  - Smaller drawdowns
  - Higher Sharpe ratios
- Quality bias
  - Significant bias towards quality firms

Outline

- 1. Challenging Markowitz
- **2. Risk parity investment strategy on equities**
- 3. Risk parity investment strategy on corporate bonds
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Portfolio construction procedure

Estimation of risk parameters

- Industry betas: Ordinary Least Squares on trailing time-window
- Specific volatilities: standard deviation of residual returns, same window,
  if data points lack, proxy by logarithm of capitalization*
- Convergence by iterative re-estimation

Portfolio construction

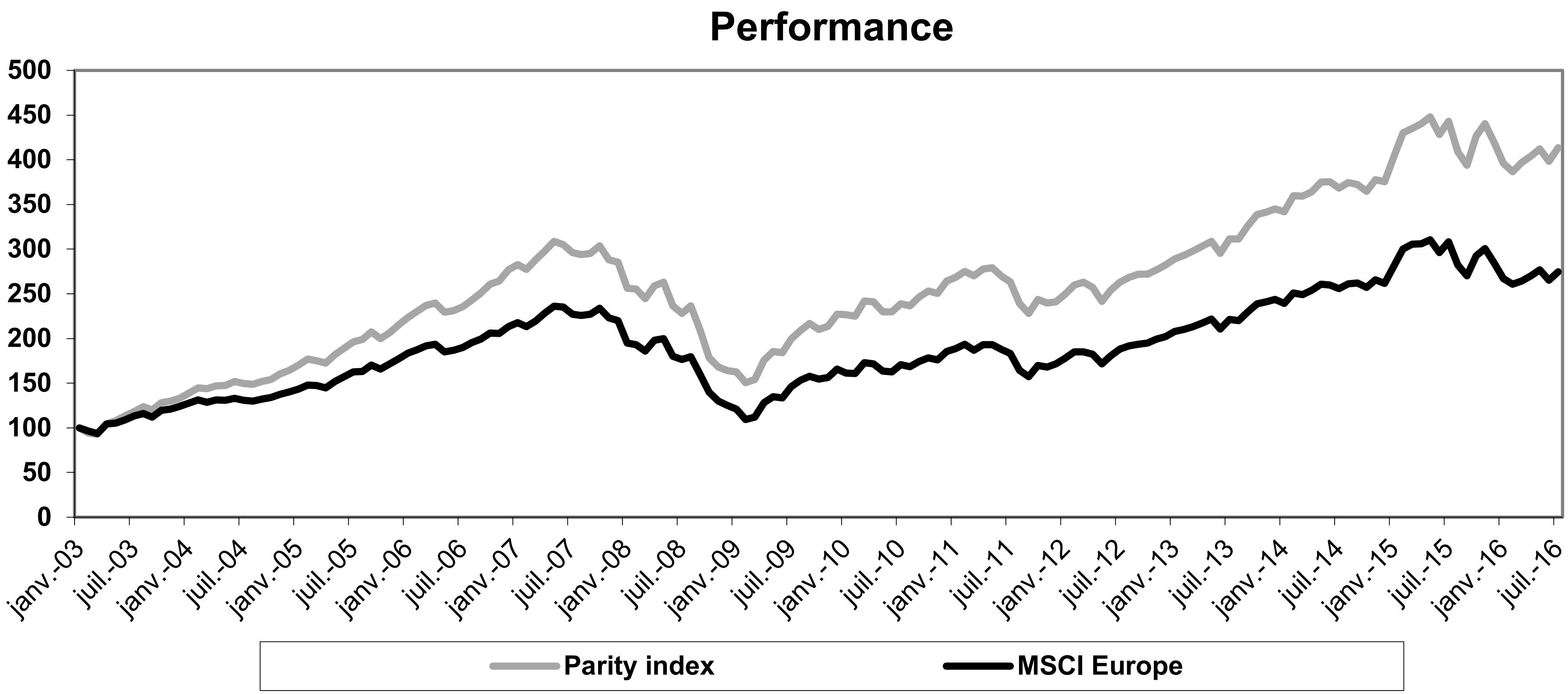
- Minimize tracking error (TE) with respect to target index
- Covariance matrix as defined by CAPM
- Constrain industry weights (to be $1/\beta$)
- Limit number of holdings
- Monthly rebalancing

Portfolio features

- ± 200 holdings out of 450

* Folllowing Barr (1976)

Equity return performance



Key figures

	MSCI Europe	Parity index
total return	7.8%	11.1%
volatility	14.3%	14.3%
Sharpe ratio	0.4	0.6
max drawdown	-54%	-51%
TE	-	2.8%
turnover per month (weight)	2.8%	2.1%

All figures are annual if not specified

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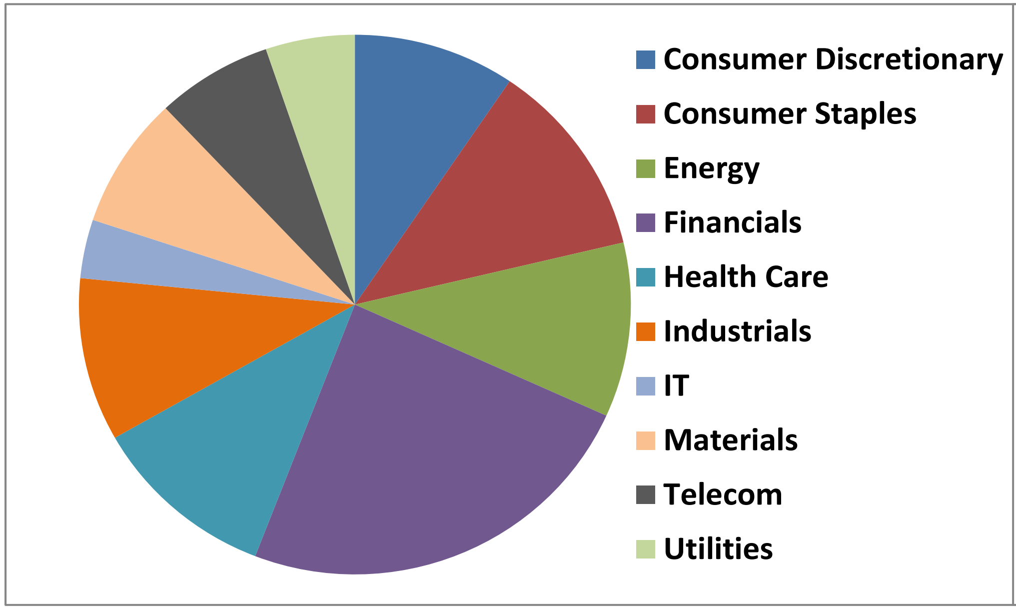
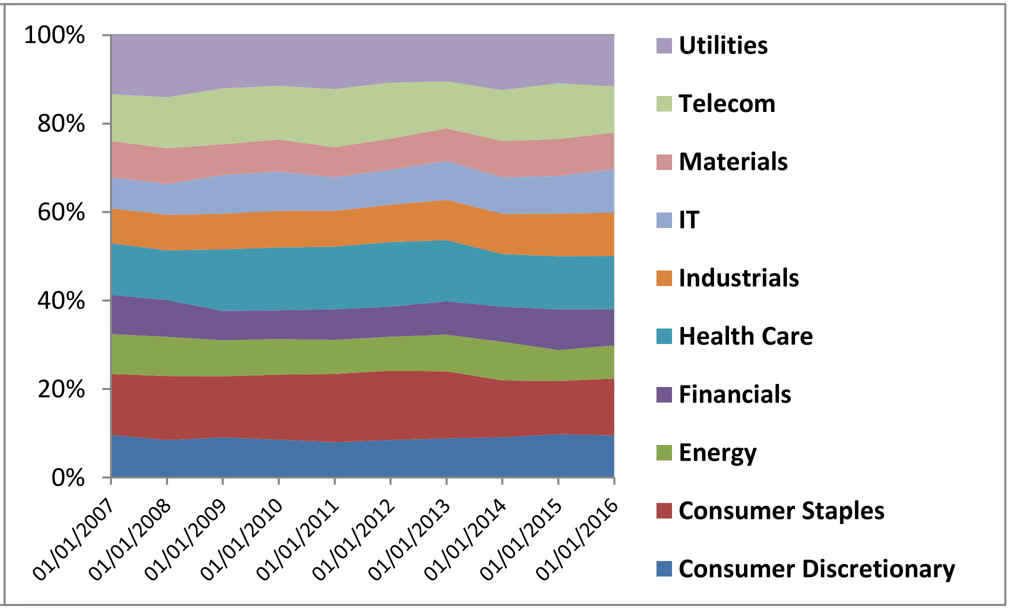
Sector parity

Starting from a non-equilibrated sector breakdown (left) risk contributions are set equal per sector (right).

market sector breakdown

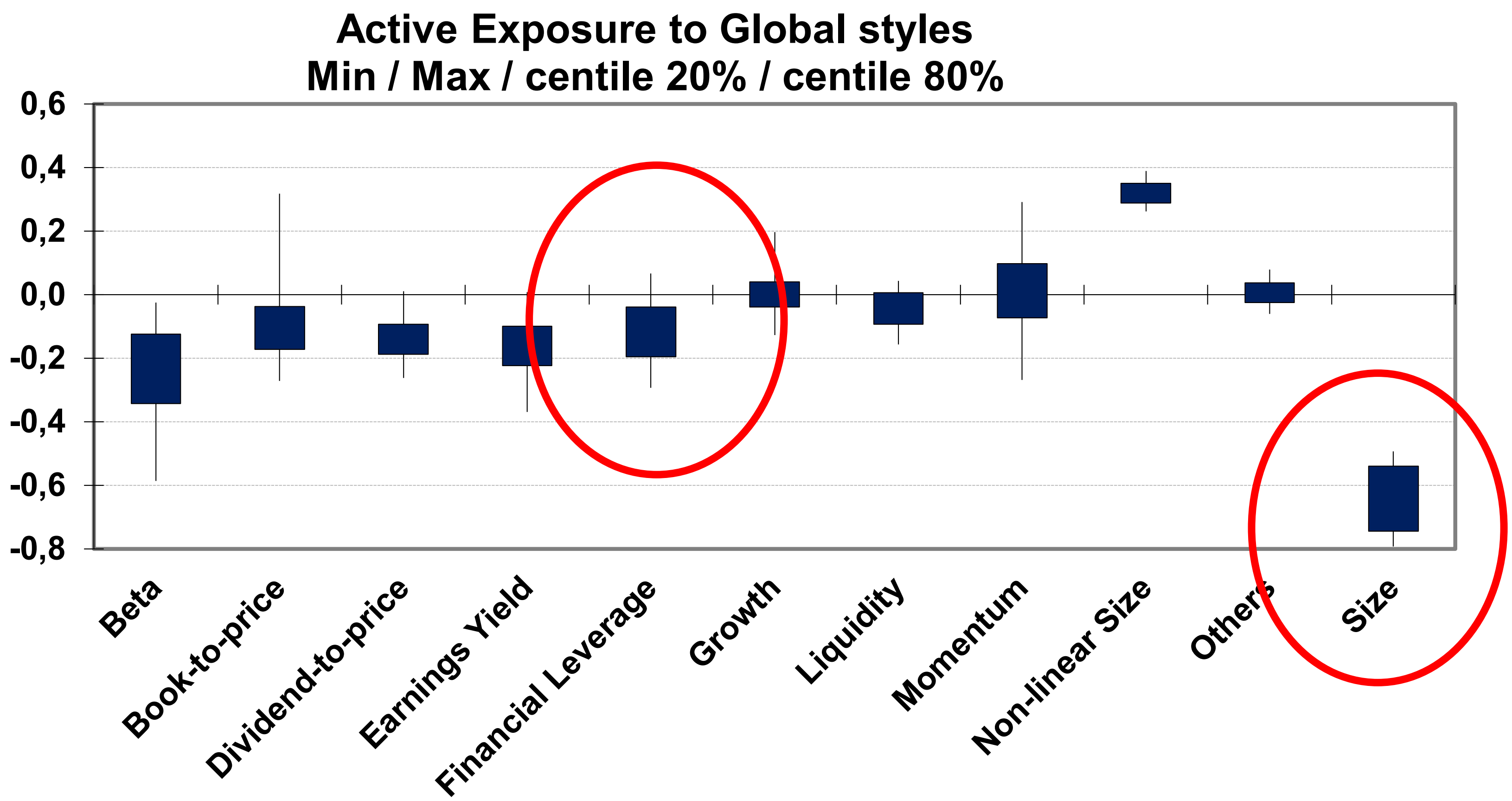


breakdown parity index over time



Induced quality & small cap bias

The parity index has a structural bias towards low-leverage firms and to small stocks.



Outline

Portfolio construction procedure

Specification of risk

- Industry betas: weighted average Duration times Spread (DTS)*
- Specific volatilities -in fact specific betas- weighted average DTS per issuer

Portfolio construction

- Through stratified sampling (match target index on DTS per industry)
- Limit number of holdings
- Quarterly rebalancing
- Add liquidity criteria to selection process, i.e. debt size and bond age
- Turnover reduction: priority given to bonds already held

Portfolio features

- ± 200 holdings out of 1840
- Duration mismatch (if any) cancelled through derivatives overlay

* Ben Dor et al. (2007)

Portfolio construction procedure cont.

Refined universe definition

– Barclays Euro Corp Index reduced to “European Large Cap version” by

  * discarding non-European issuers

  * discarding third smallest capitalization

Industry regrouping

– 16 Barclays lvl3 sectors aggregated to 10 groups of comparable caliber

  * Financial companies regrouped with banks

  * Leisure regrouped with Industry

  * Banks split into senior and subordinated debt

Firm level

– Firms’ debt structure conserved

  * Sub-selection of bonds sought to represent the firms

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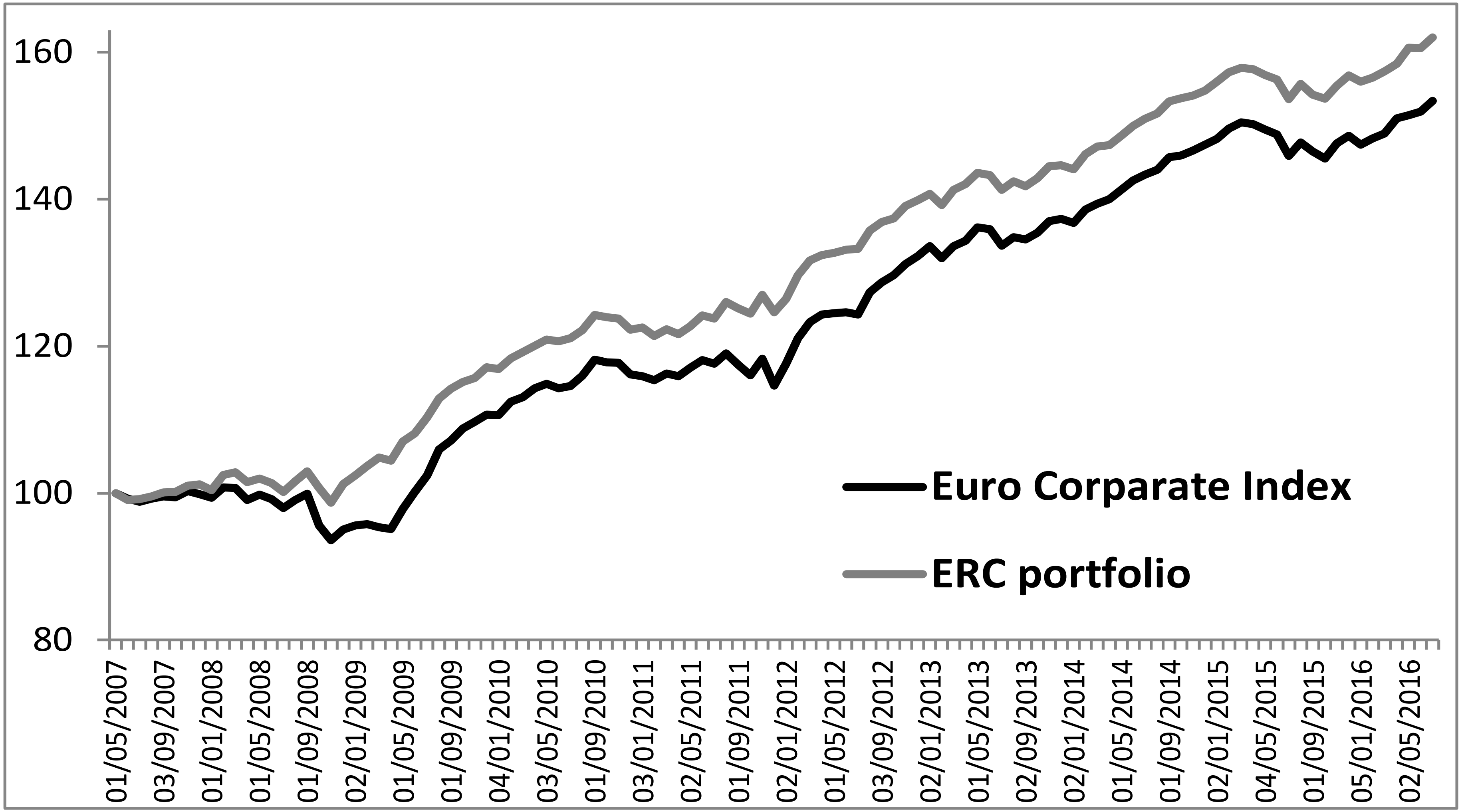
Return performance



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Key figures

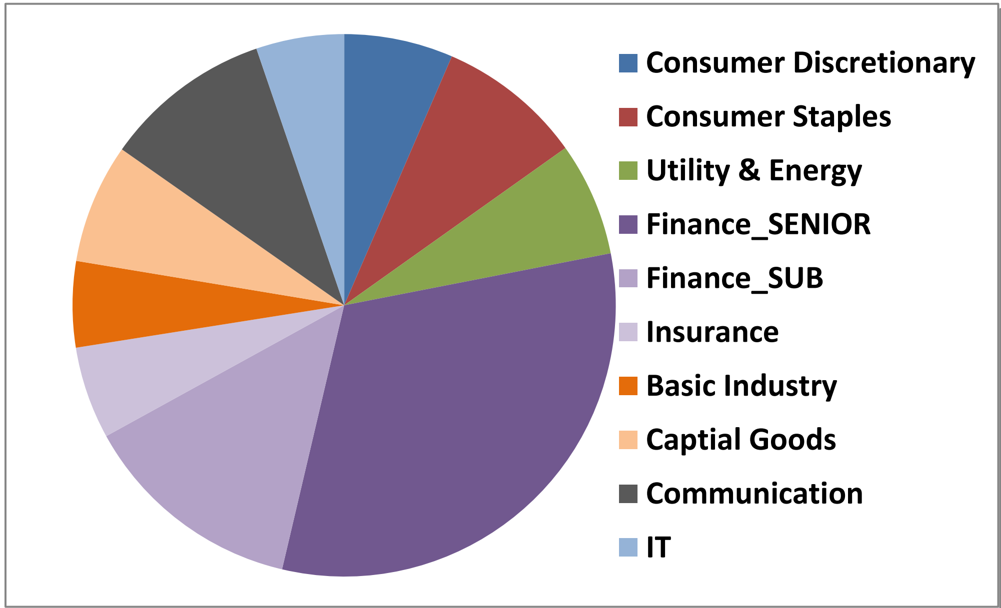
	benchmark	Parity index	Parity portfolio
TTR	4.8%	5.4%	5.4%
volatility	4.1%	3.4%	3.4%
Sharpe ratio	1.2	1.6	1.6
max drawdown	-6.3%	-4.3%	-4.1%
TE	-	1.64%	0.11% (from target)
# bonds	1843	1843	≤ 233
# issuers	400	400	≤ 127
turnover per month #	30	30	10
weight	0.3%	-	1.6%

All figures are annual if not specified

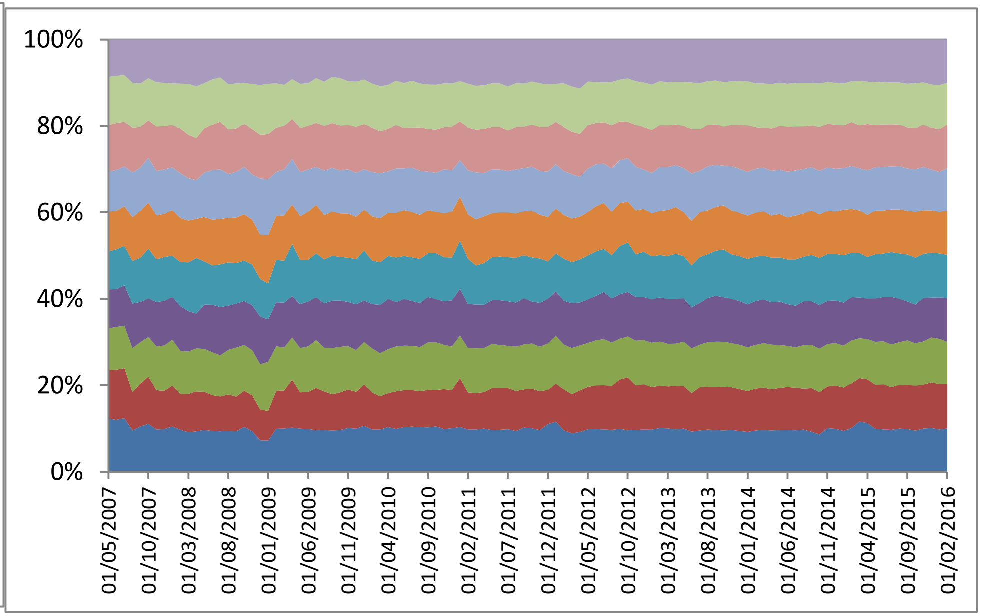
Sector parity

Starting from a non-equilibrated sector breakdown (left) risk contributions are set equal per sector (right).

market sector breakdown



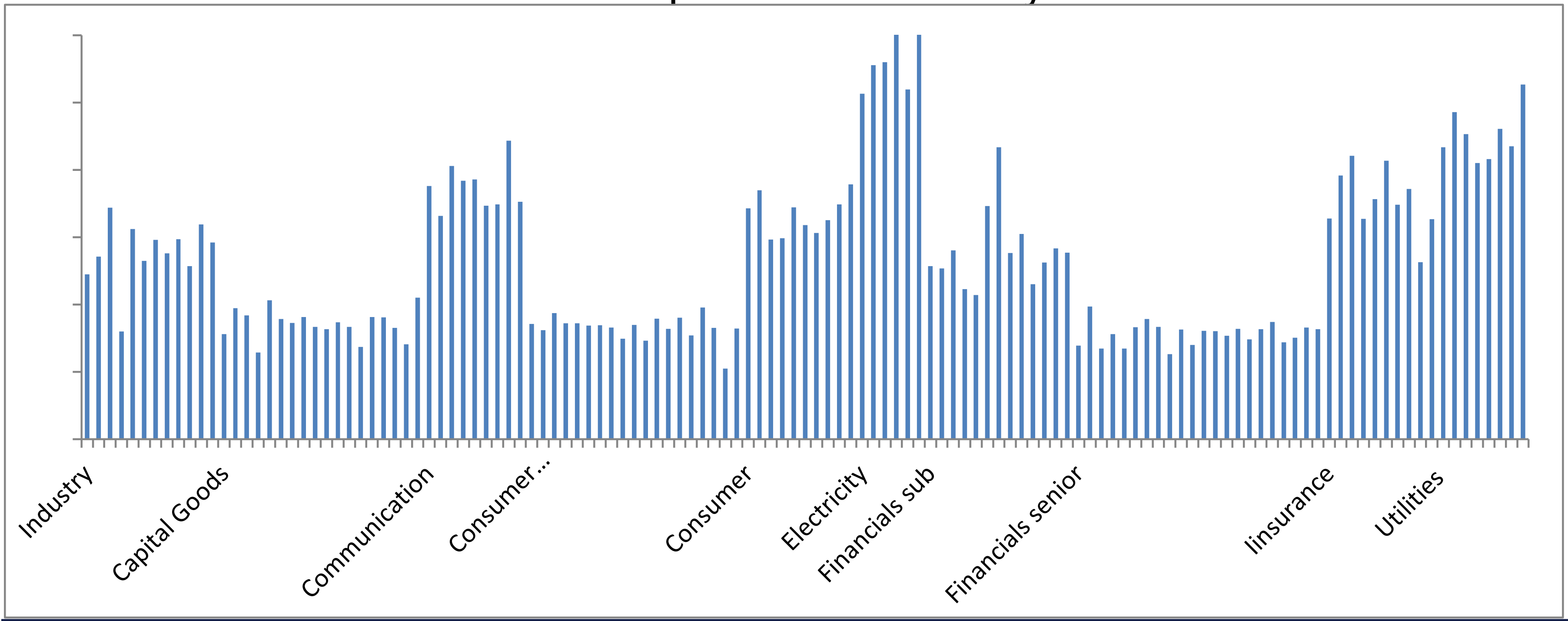
sector risk contributions over time



Risk breakdown of the portfolio

Despite the limited number of holdings (230 out of 1800) and the turnover- and liquidity constraints, the risk parity is more or less respected.

Risk contribution per firm ordered by sector



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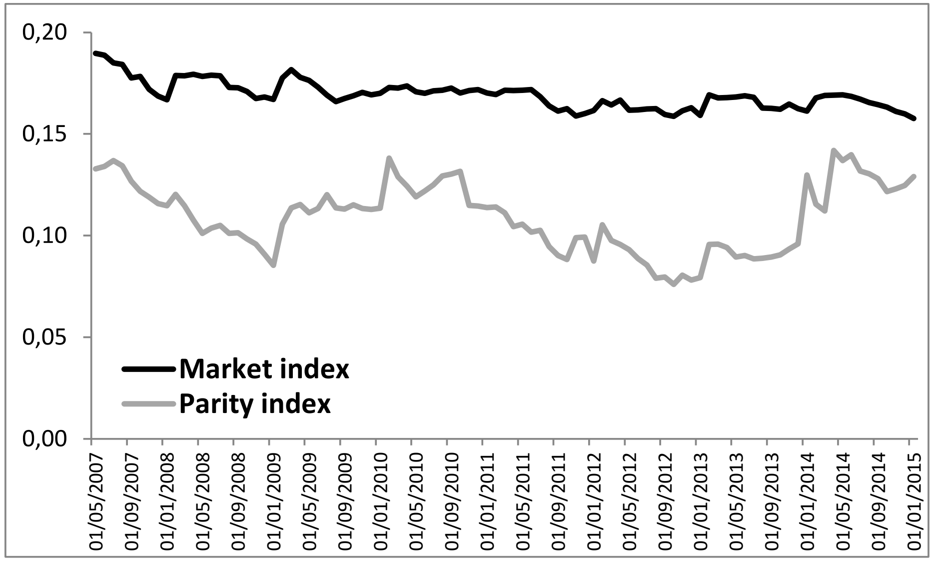
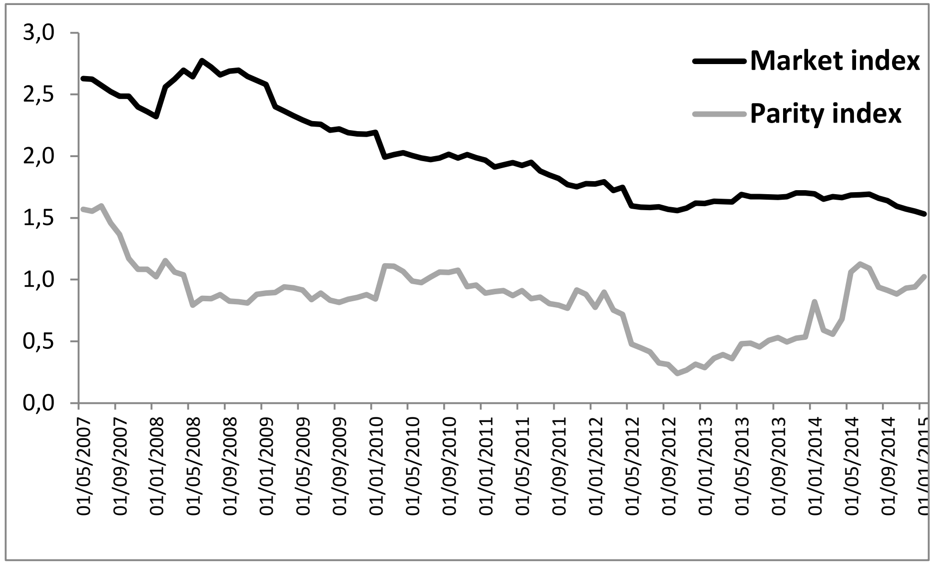
Induced quality bias

The parity index has a structural bias towards low-indebted firms
As measured by two debt ratios.

Debt to assets



Debt to equity



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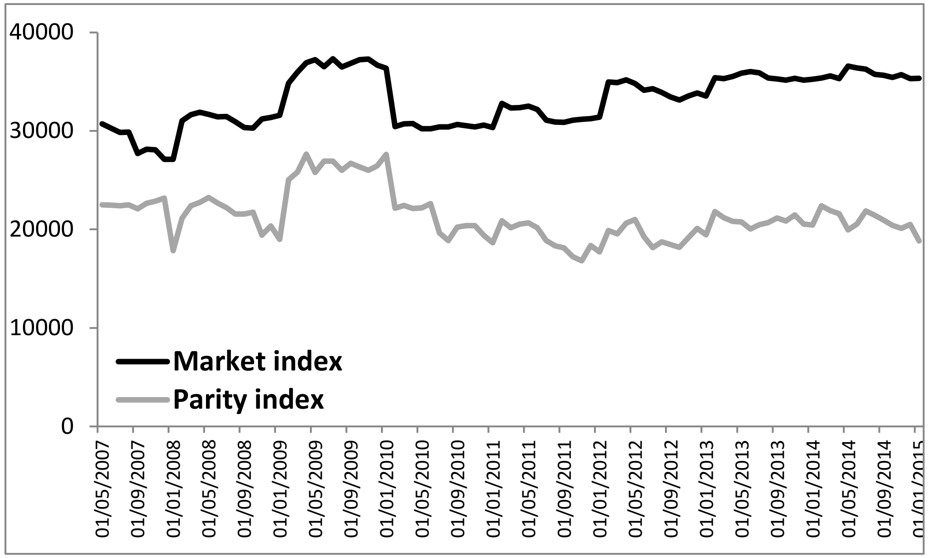
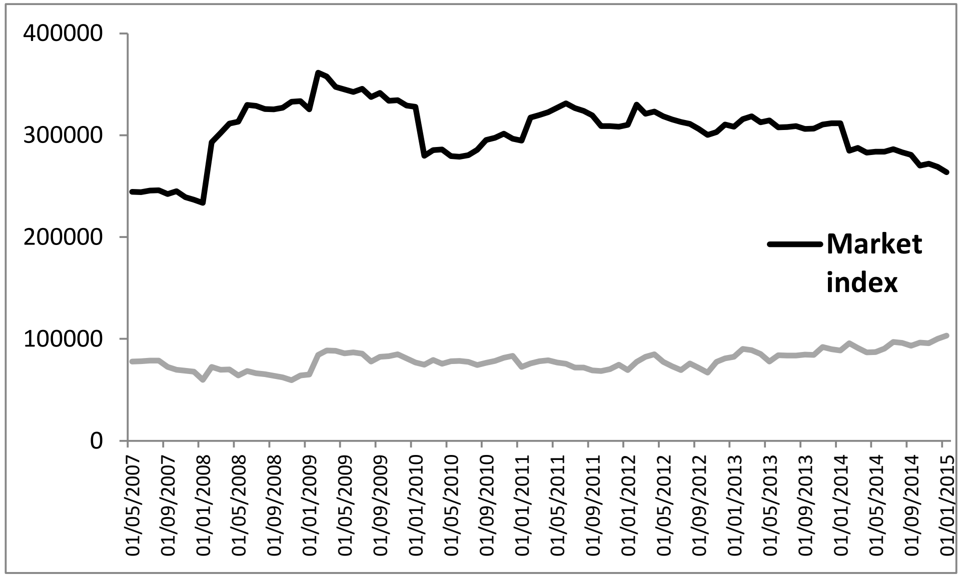
Induced size bias

The parity index has a structural bias towards small firms as measured by two firm fundamentals, sales revenues and book value

Sales



Assets



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Conclusion

The wave of *smart* investing strategies brings new vision on investment management, placing Modern Portfolio Theory in perspective.

Robust risk parity is akin to robust investing, shrinkage methods and stability-adjusted portfolios.*

Our conviction is that the market factor constitutes risk with ‘certainty’. For any other factors trading off safety for risk efficiency is debatable.

Interestingly risk-parity investing incites to holding long-only portfolios.

* Kritzman and Turkington (2016)

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Coherent risk parity investing*

Setting 2bis. constrained version of CAPM holds

$$\forall_{i \in industry\ j}: \quad R_{it} = \beta_j \cdot {R_t}^M + \varepsilon_{it}$$

Optimum when $x_i = {}^1/_{\sigma_i}$ between individual assets

$x_j = {}^1/_{\beta_j}$ between industry groups

Our conviction

The market factor should not be ignored in a portfolio optimisation. It specifies asset price behaviour with quasi-certainty. This can be concluded on the basis of the extremely high T-statistics for beta estimates, ±25.

For additional factors trading off safety for risk efficiency is debatable.

* Russo (2014)

Appendix

Theorem

Setting the industry weights inversely proportional to the betas equalizes risk contributions

Proof

Let $\forall_{i \epsilon j}: \; R_{it} = \beta_j \cdot R_t^M + \varepsilon_{it}$ $\qquad \varepsilon_i \sim n(\,0,\; \sigma_i^2\,)$

then

$$R_{jt} = \beta_j \cdot R_t^M + \sum_i {}^{\varepsilon_{it}}/_{\sigma_i} \qquad \sum_i {}^{\varepsilon_{it}}/_{\sigma_i} \sim n(0{,}1)$$

Risk Contribution (RC) equal if $x_j = {}^{1}/_{\beta_j}$

$$RC_1 = x_1^2\,\beta_1^2 + x_1 x_2 \beta_1 \beta_2 + 1$$
$$RC_2 = x_2^2\,\beta_2^2 + x_1 x_2 \beta_1 \beta_2 + 1$$