Risk Stabilization and Asset Allocation

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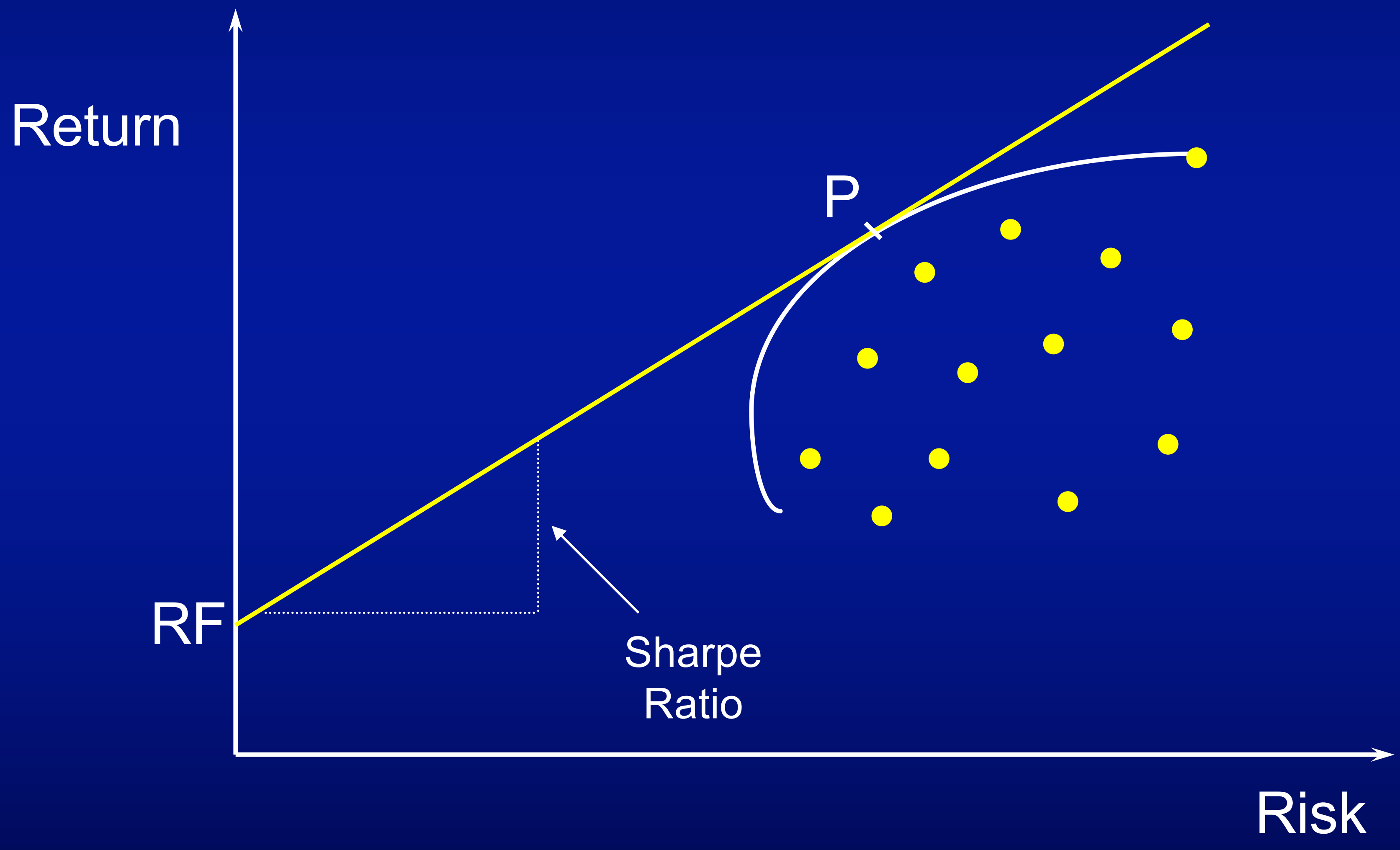
The Purpose of Asset Allocation

- To make efficient tradeoffs between broad asset class risks and expected returns

Efficient Frontier



Traditional Approach to Asset Allocation

- Primarily returns-based
- Example:
  - Desired return: Inflation + 4%
  - Equities: Inflation + 6%
  - Fixed Income: Inflation + 1%
  - Implied allocation: 60/40
- Risk is secondary
- Long-run historical estimates
- Policy portfolios with static weights

Time Varying Stock Market Volatility (VIX)

Changing Stock-Bond Correlation
(U.S., Trailing 36 months, 1950 to 2015)

Time Varying Risk

- When asset class risks and correlations are time varying, the risk of a static allocation is also time varying
- A static allocation thus is a very noisy representation of portfolio risk

Stable Risk Policies

- Vary the asset allocation to maintain stable portfolio risk
  - Reduce the exposure to global equities when equity market risk is high
  - Increase the exposure to global equities when equity market risk is low
- Basic questions:
  - Is risk forecastable?
  - What is the relationship between risk and expected return?

Theoretical Relationship Between Expected Return and Risk

- Optimal exposure

  = Risk Tolerance x Risk Premium/Variance

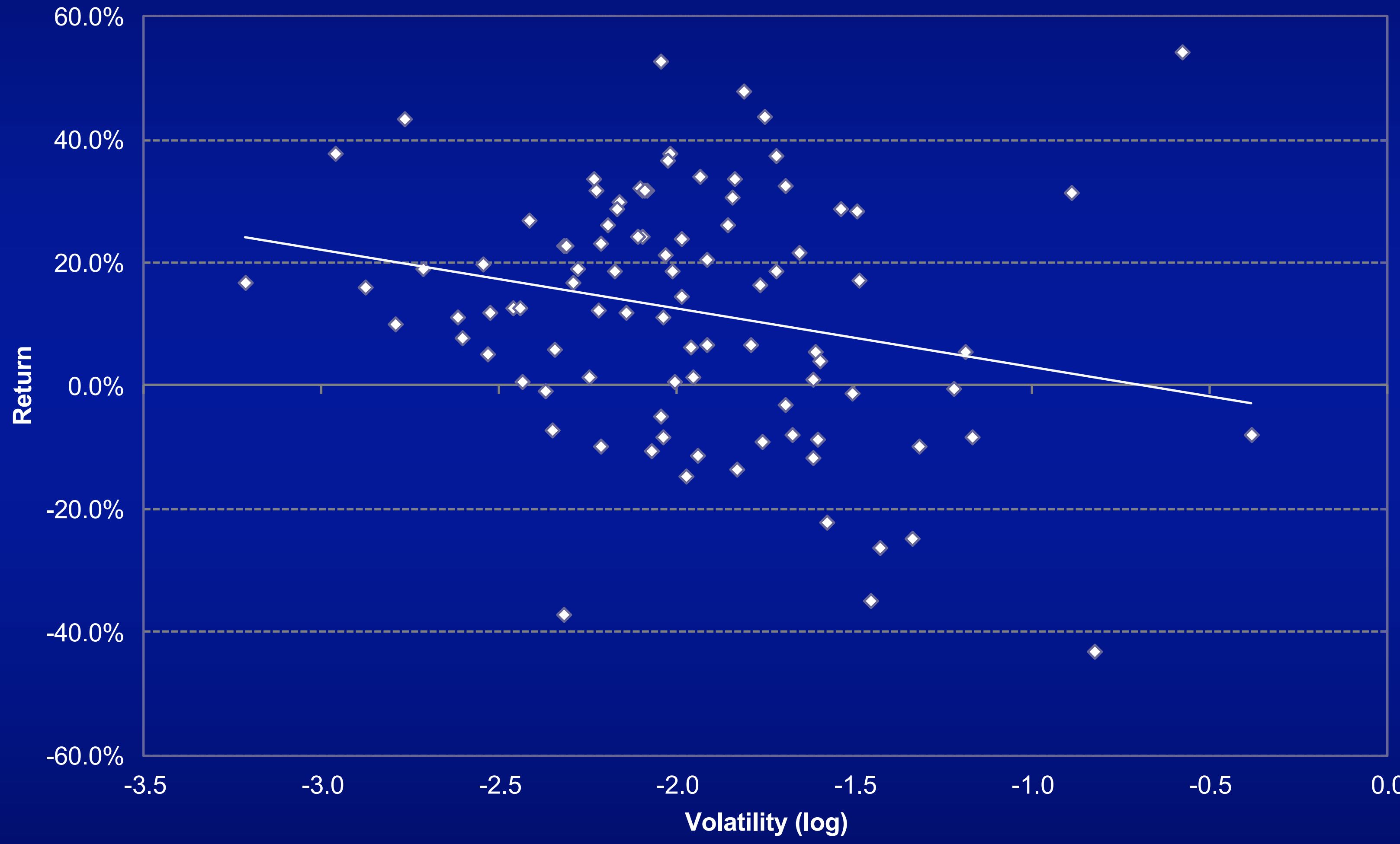
- In equilibrium, exposure = constant

- Theoretically, therefore

  Risk premium = proportional to variance

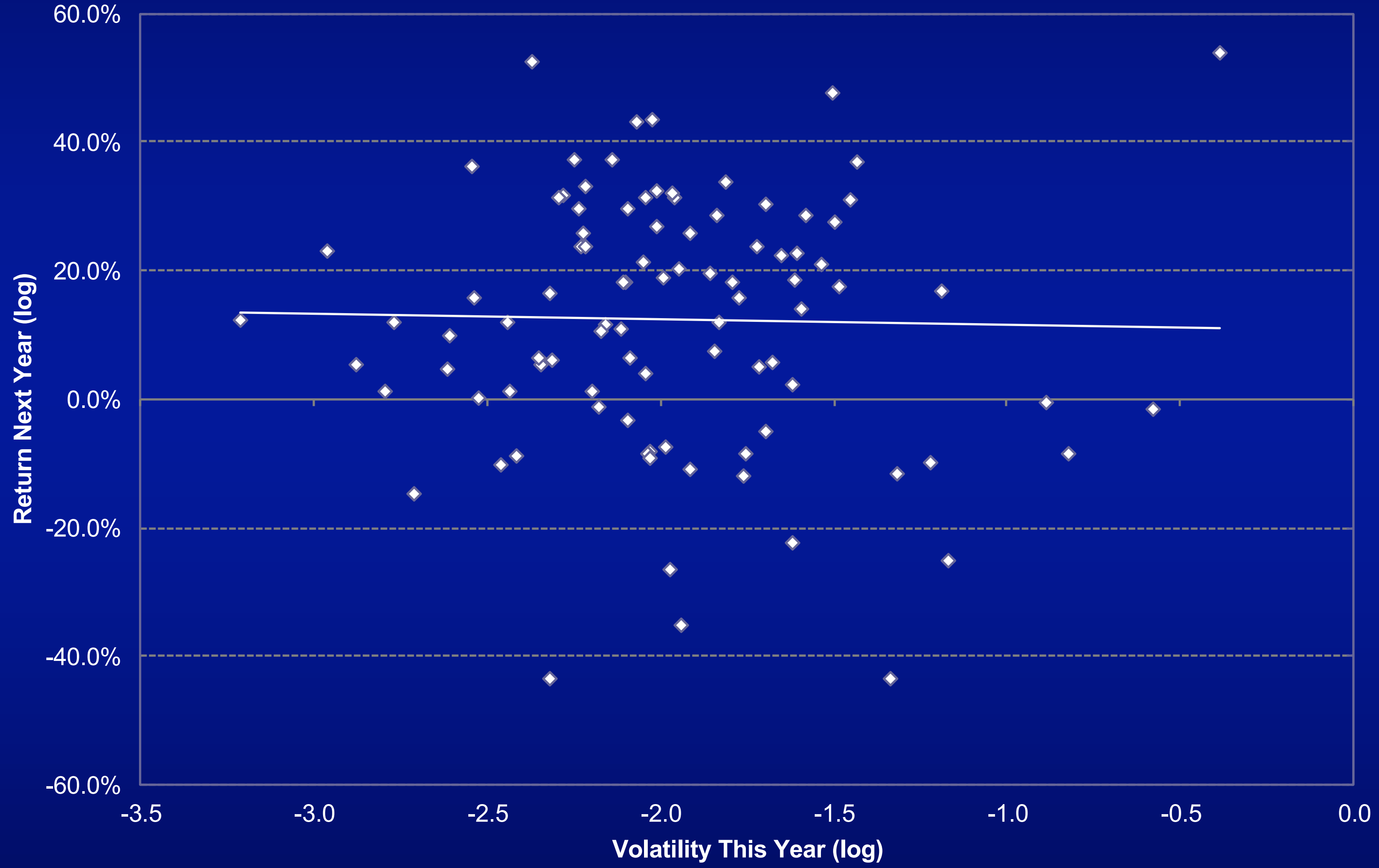
U.S. Equities: Vol vs Contemp. Return

(Intra-Year Volatility, 1920-2015)



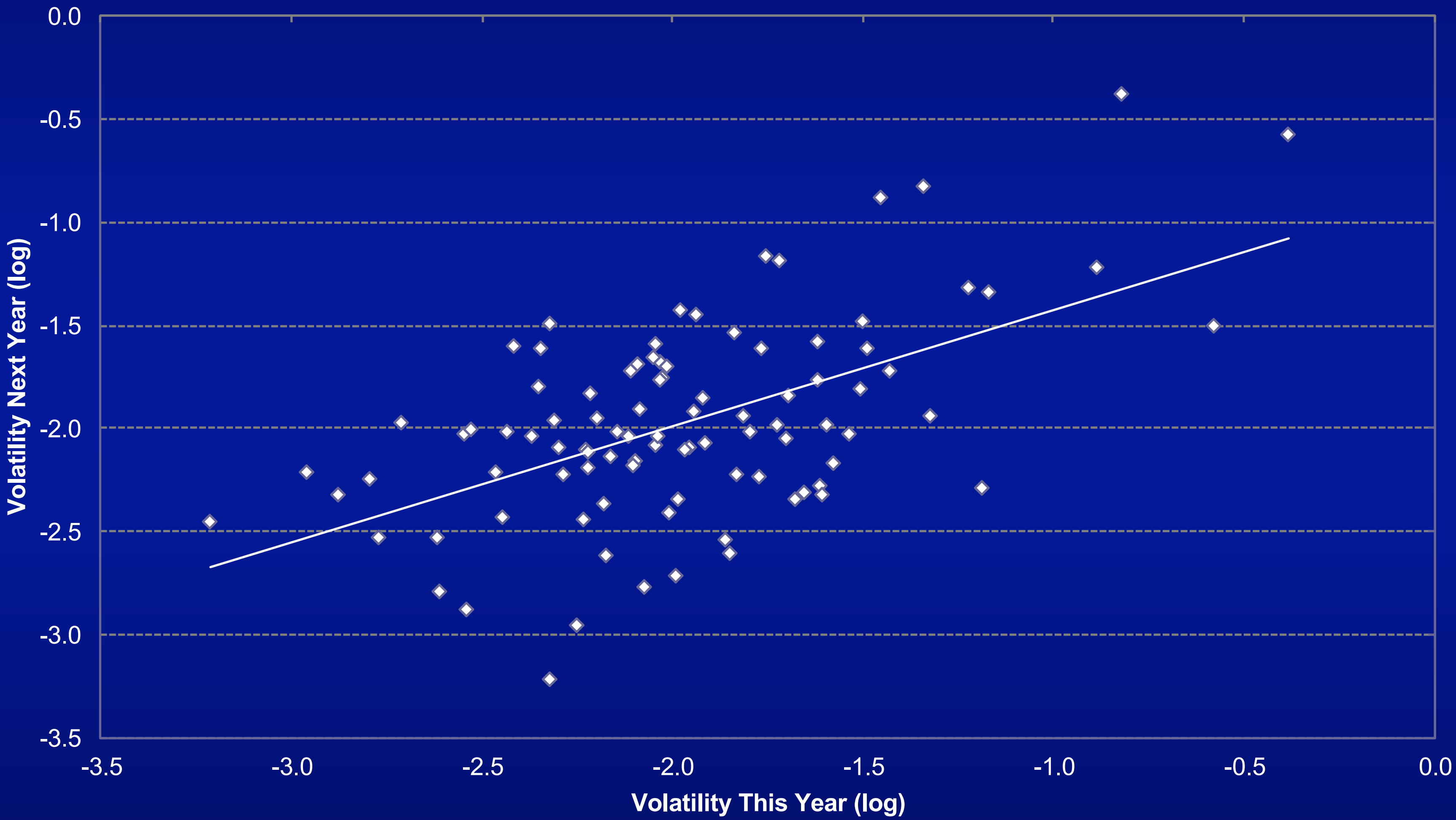
U.S. Equities: Volatility vs Future Return

(Intra-Year Volatility, 1920-2014)



Persistence in Volatility

(Intra-Year Volatility, 1920-2015)



Stylized Facts

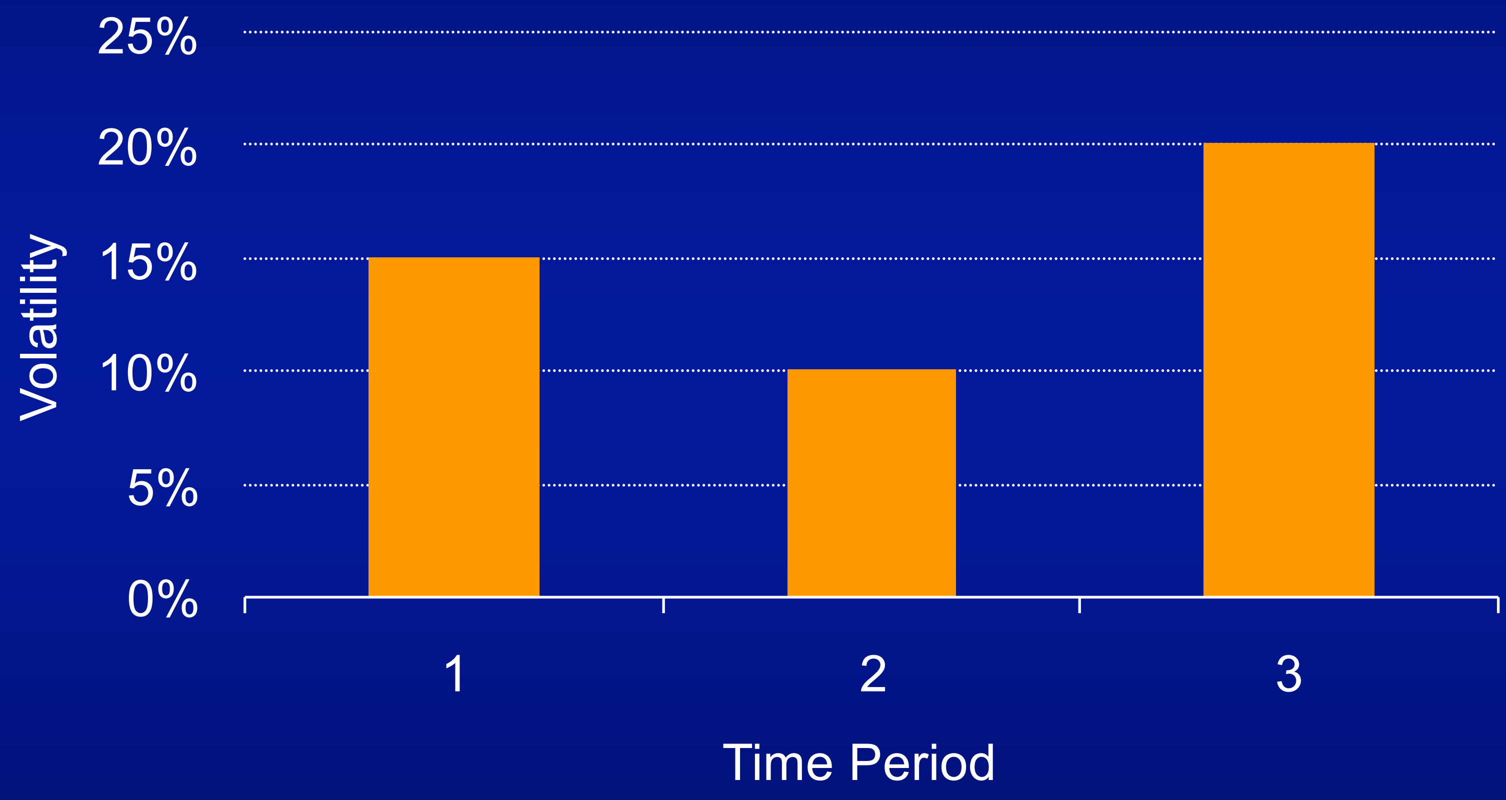
- Volatility varies significantly
- Volatility is predictable
- Volatility predicts risk
- Volatility seems only weakly related to expected return

Volatility Predicts Risk But Not Return

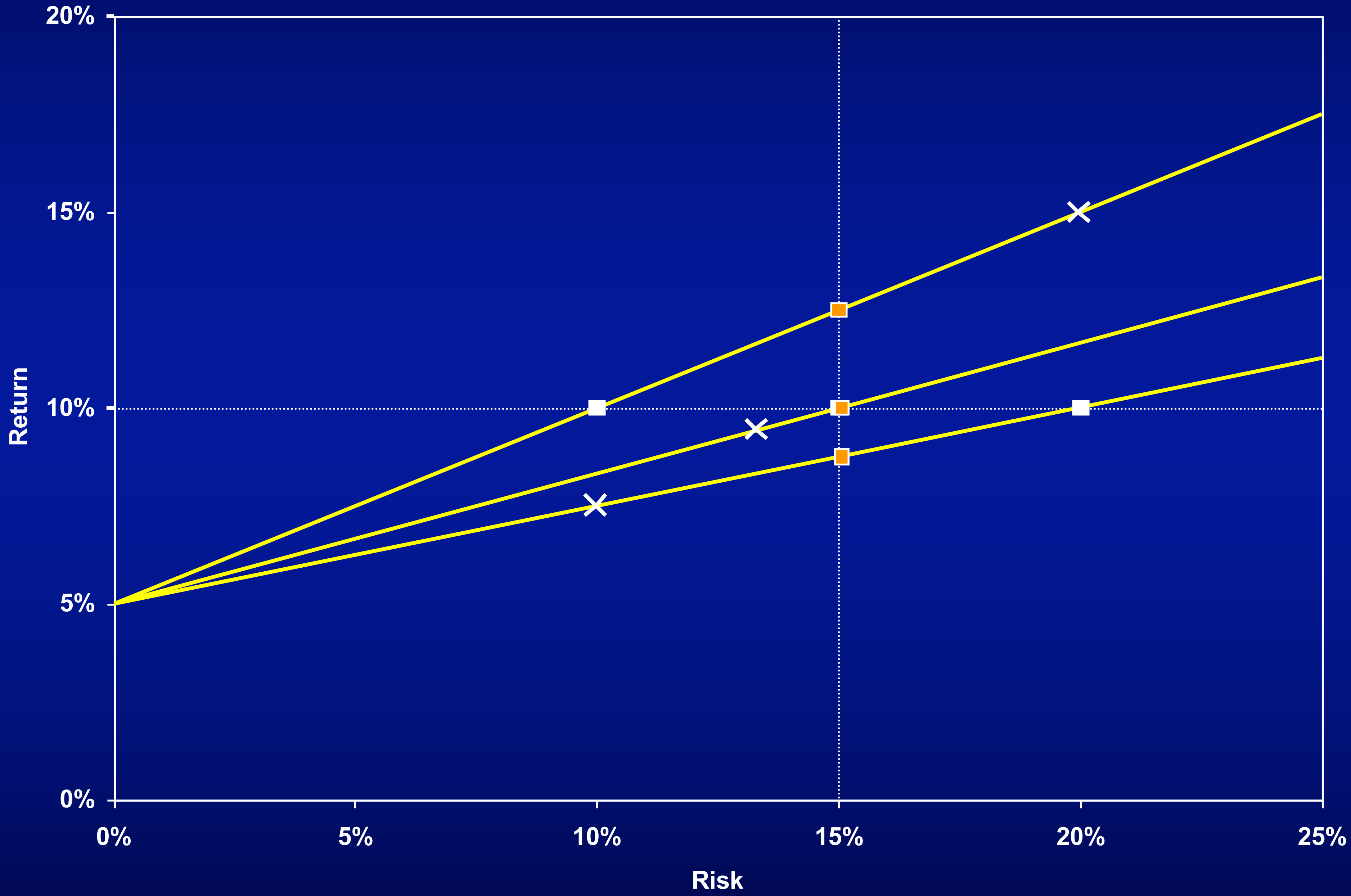
(S&P 500, 1926-2014)

Past 12 Months	Next 12 Months		
Regime	Average Bad Return (lower 25%)	Average Normal Return (middle 50%)	Average Good Return (upper 25%)
Low Volatility (below 11%)	-5.7%	13.2%	33.3%
Medium Volatility (11-16%)	-11.6%	14.1%	36.7%
High Volatility (above 16%)	-23.6%	8.9%	37.2%

Changing Volatility: An Example



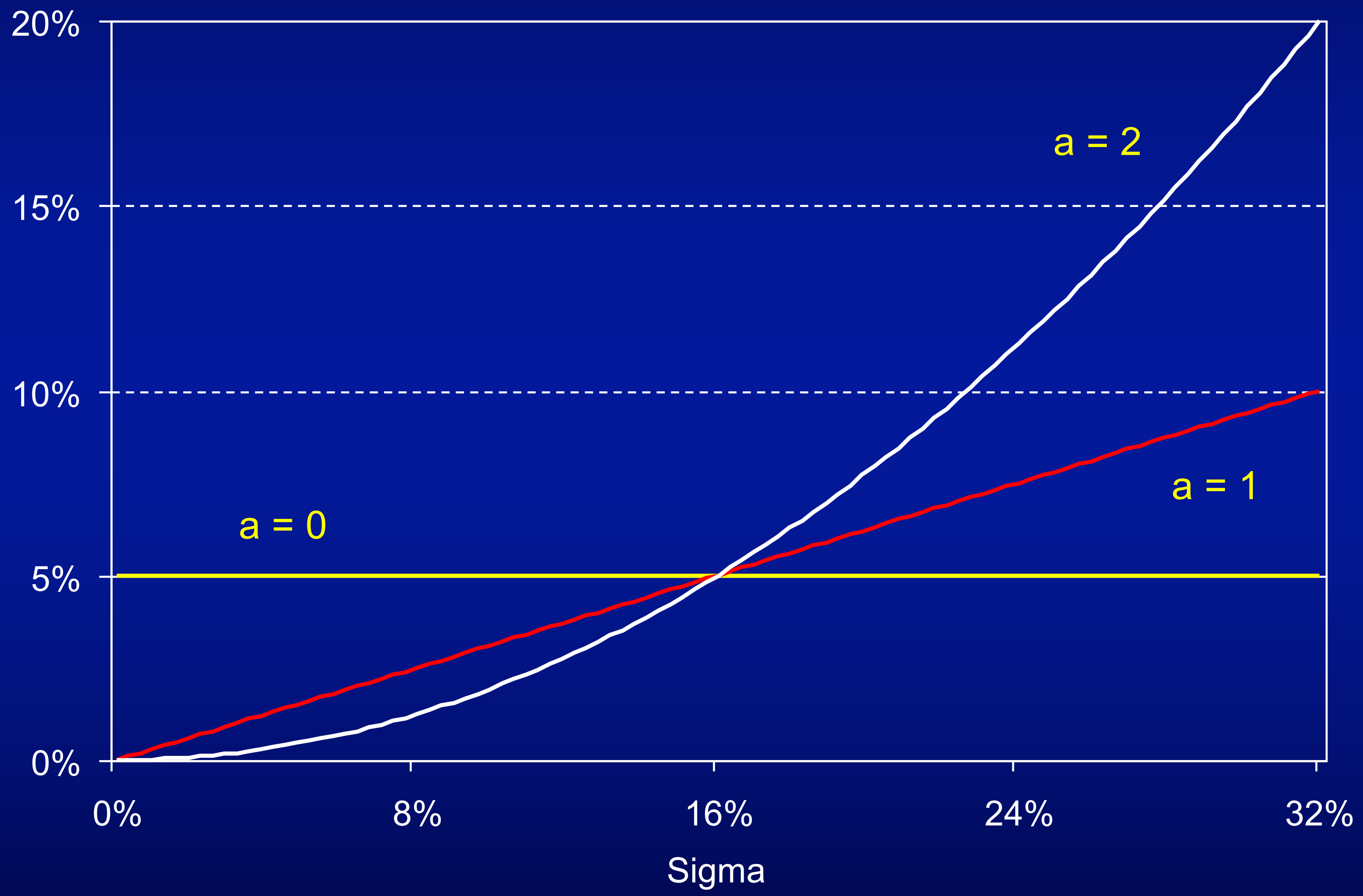
Changing Volatility (cont.)



The Model

- Allocation x_t to a risky portfolio and $1\text{-}x_t$ to a riskless asset
- Risk Premium is proportional to σ_t^a
  - a = 2: Proportional to variance
  - a = 1: Proportional to standard deviation
  - a = 0: No relation to risk

Risk Premium vs. Sigma



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- Log σ_t is normally distributed with variance = v^2
- σ'_t is a forecast of σ_t with correlation b

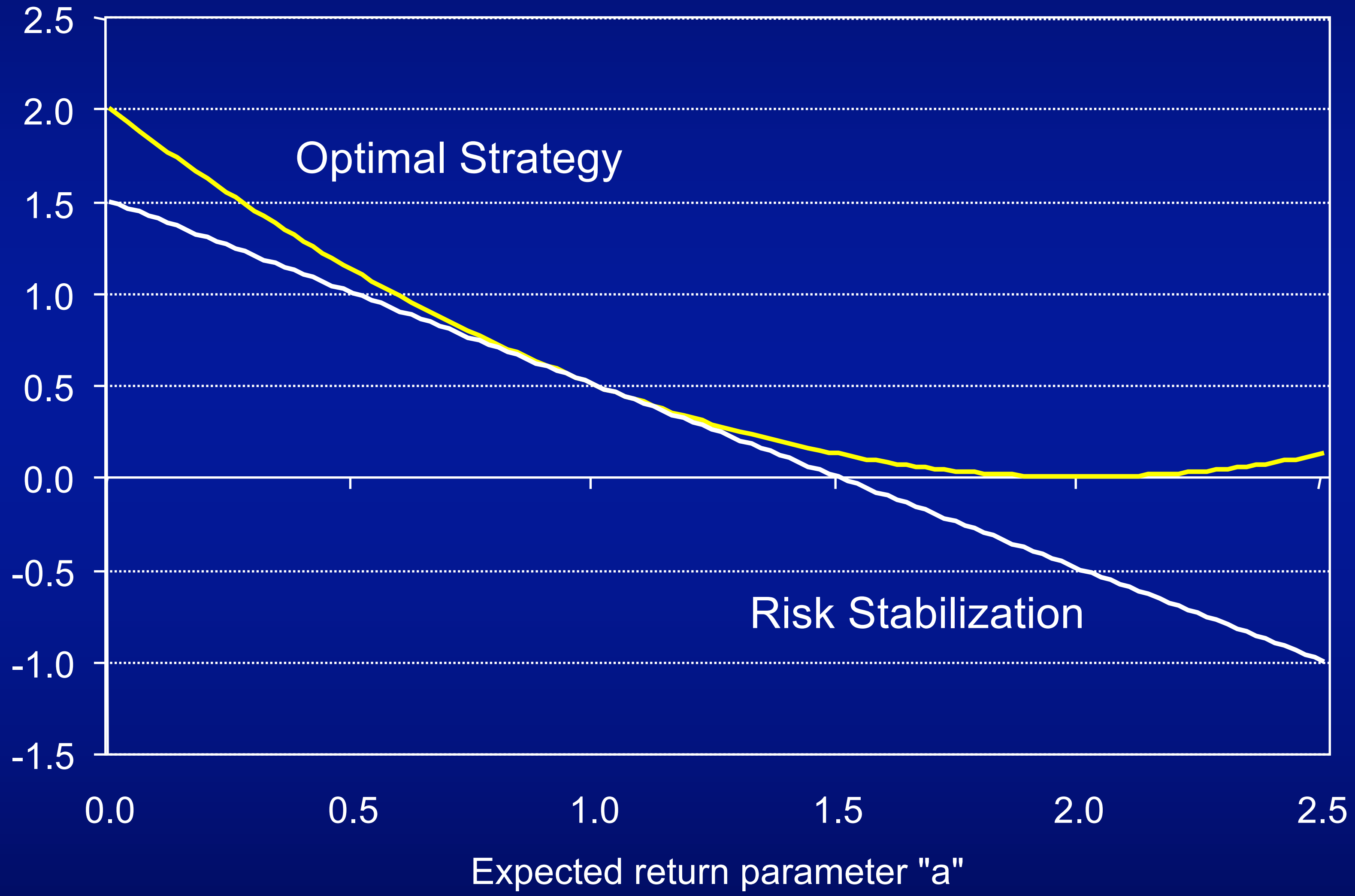
Strategies

- Static policy allocation
  - X_t = constant
- Stable risk policy
  - $X_t = \sigma^*/\sigma'_t$
- Optimal strategy
  - $X_t = \text{constant}/\sigma'^{(2-a)}_t$

Main Finding

Policy	Sharpe Ratio
Stable Allocation	S
Optimal Strategy	$S \times \exp \frac{1}{2}(a-2)^2 b^2 v^2$
Stable Risk	$S \times \exp (3/2-a) b^2 v^2$

Sharpe Ratio Improvement



Economic Significance

v	b	Static Allocation Sharpe Ratio	Sigma
0.5	0.8	0.5	10%

	Increase in Sharpe Ratio		Increase in Portfolio Expected Return	
a	Optimal Strategy	Stable Risk	Optimal Strategy	Stable Risk
0.0	0.38	0.27	1.89%	1.36%
0.5	0.20	0.17	0.99%	0.87%
1.0	0.08	0.08	0.42%	0.42%
1.5	0.02	0.00	0.10%	0.00%
2.0	0.00	-0.08	0.00%	-0.38%

Kurtosis

Policy	Sharpe Ratio
Stable Allocation	K
Optimal Strategy	$K \times \exp \{(a-1)^2-1\} 4b^2v^2$
Stable Risk	$K \times \exp -4b^2v^2$

Risk Stabilized ACWI

Asset allocation with time varying risk

- Static Policy Allocation
  - Random risk
  - Lowest Sharpe Ratio (if returns vary slowly with risk)
  - Highest kurtosis
- Stable Risk Policy
  - More precise risk
  - Random exposure
  - Average Sharpe Ratio
  - Don't need knowledge of expected return
  - Lowest kurtosis
- Optimal Strategy
  - Random risk and exposure
  - Highest Sharpe Ratio
  - Need knowledge of expected return
  - Intermediate kurtosis